O Level A Maths Tutorial 7: Trigonometric Functions, Identities and Equations

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Syllabus :

- Six trigonometric functions for angles of any magnitude (in degrees or radians).

1.

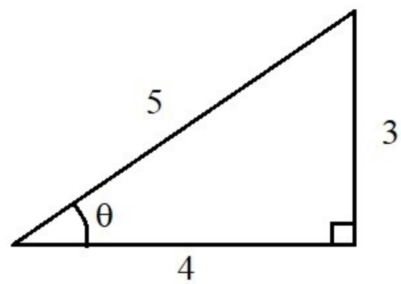

Figure 7-1

Write down the values of :

(i) $\sin\theta$	(iv) $\operatorname{cosec}\theta$
(ii) $\cos\theta$	(v) $\sec\theta$
(iii) $\tan\theta$	(vi) $\cot\theta$

- Principal values of $\sin^{-1}x$, $\cos^{-1}x$, $\tan^{-1}x$

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2. Find the principal values of :

(i) $\sin^{-1} \frac{1}{2}$	(iv) $\cos^{-1} \sqrt{3}/2$
(ii) $\sin^{-1} \sqrt{3}/2$	(v) $\tan^{-1} 1$
(iii) $\cos^{-1} \frac{1}{2}$	(vi) $\tan^{-1} 1/\sqrt{3}$

• Exact values of the trigonometric functions for special angles (30°, 45°, 60°) or (π/6, π/4, π/3)

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3. (i) Using Pythagoras, find the height h.

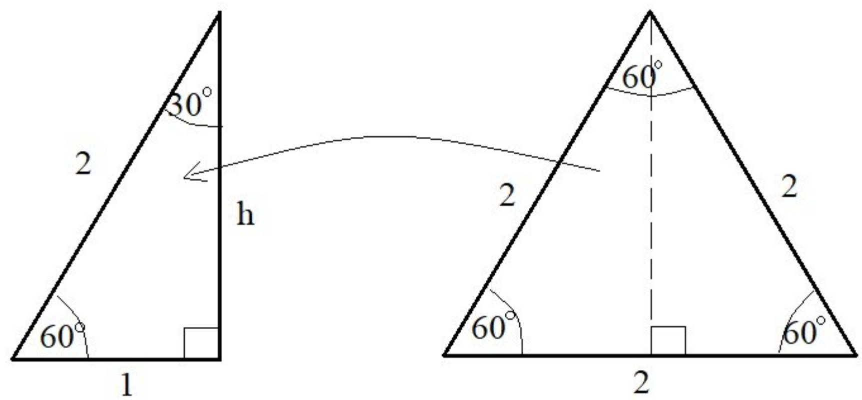

Figure 7-2

(ii) Then find (without calculator) the values of cos 60°, sin 60° and tan 60°.

(iii) And also find (without calculator) the values of cos 30°, sin 30° and tan 30°.

4. (i) Describe how the angle of 1 radian is defined.

(ii) Derive, in terms of π, the value of 1° in radians

(iii) Derive, in terms of π, the value of 30°, 45°, 90°, 180° and 360° in radians

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• Amplitude, periodicity and symmetries related to sine and cosine functions

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5. State the amplitude, periodicity and symmetry of

(i) $y = \sin x$
(ii) $y = \cos x$

- Graphs of y = a sin (bx) + c, y = a sin (x/b) + c, y = a cos (bx) + c, y = a cos (x/b) + c and y = a tan (bx), where a is real, b is a positive integer and c is an integer.

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6. Sketch the following graphs for $0 \le x \le 360°$.

(i) $y = 2 \sin x$

(ii) $y = \cos x + 1$

(iii) $y = \frac{1}{2} \sin 2x$

(iv) $y = 2 \cos (x/2) - 2$

(v) $y = \tan x$

(vi) $y = \tan (x/2) + 1$

• Use of:

- $\frac{\sin A}{\cos A} = \tan A$, $\frac{\cos A}{\sin A} = \cot A$, $\sin^2 A + \cos^2 A = 1$,

  $\sec^2 A = 1 + \tan^2 A$, $\text{cosec}^2 A = 1 + \cot^2 A$

- the expansions of $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$

- the formulae for sin 2A, cos 2A and tan 2A

- the expression of $a \cos \theta + b \sin \theta$ in the form $R\,cos(\theta \pm \alpha)$ or $R \sin(\theta \pm \alpha)$

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7. (a) Using Pythagoras theorem, prove that $\sin^2 A + \cos^2 A = 1$.

(b) Given $\sec A = 1 / \cos A$, prove that $\sec^2 A = 1 + \tan^2 A$.

(c) Given $\text{cosec } A = 1 / \sec A$, prove that $\text{cosec}^2 A = 1 + \cot^2 A$.

8. Prove this compound angle formula

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

using this figure.

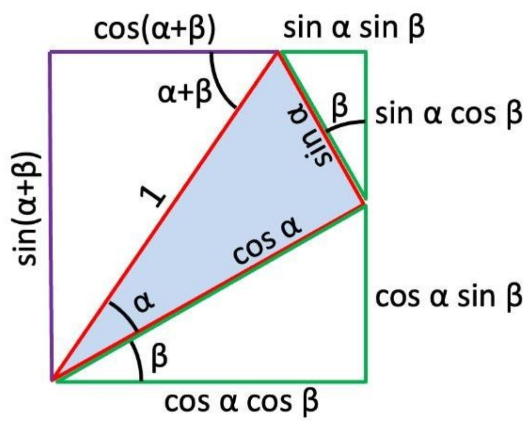

Figure 7-3

9. Using the above figure, prove

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

10. (a) Explain why

$$\sin(-x) = -\sin x$$

and

$$\cos(-x) = \cos x.$$

(b) Using the results from questions 8, 9 and 10(i), obtain the compound angle formulae for

(i) sin(A - B)

(ii) cos(A - B)

11. Using the resu;ts from questions 8 to 10, obtain the expressions for :

(i) tan(A+B),

(ii) tan(A – B).

12. Using the results from questions 8 to 11, obtain the expressions for :

(i) sin 2A in terms of sin A and cos A,

(ii) cos 2A in terms of sin A and cos A, and

(iii) tan 2A in terms of tan A.

13. Express $3 \cos \theta + 4 \sin \theta$ in the form $R \sin(\theta + \alpha)$. Use the formula $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

- Simplification of trigonometric expressions
- Solution of simple trigonometric equations in a given interval (excluding general solution)

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14. Find the solutions for each of the following in the given interval :

(a) $\sin x = \frac{1}{2}$ $\quad 0° < x < 180°$

(b) $\cos x = \frac{1}{2}$ $\quad 0° < x < 360°$

(c) $\tan x = \frac{1}{2}$ $\quad 0° < x < 360°$

- Proofs of simple trigonometric identities

- Refer to questions 7 and 8.

- Using trigonometric functions as models

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15. A small wooden block is attached to an elastic string. The string is hung on a support. The block is at rest.

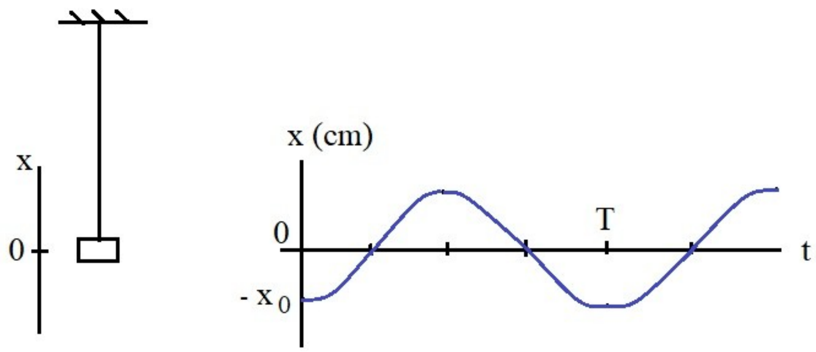

Figure 7-4

The wood is pulled downwards by 2 cm and released. The displacement x is plotted against time in the graph above. Let the amplitude be x_0 and period be T. The time for one oscillation is 2 s.

(i) Write down an equation in terms of variables in the figure to model the displacement with time.

(ii) Derive the formula for velocity as a function of time. State the maximum velocity.

(iii) Sketch two periods of the velocity time (v-t) graph. What is the phase difference from the x-t graph above?

(iv) Derive the formula for the acceleration as a function of time. State the relation between acceleration and displacement for this motion.